A Remesh-free Finite-element Method for Large Geometrical Variations and its Application to Electric Machine Design

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In this report, the methodology of an overlapping remesh-free finite element method (FEM) is presented. It avoids the disadvantage in traditional FEM that the mesh needs to be rebuilt when large geometrical variations happen. The proposed remesh-free FEM has two lays of meshes, and applies polygon elements in FEM formulation. It has the advantage of simple algorithm and is easy to be implemented in computer program. Numerical experiments have validated that the overlapping remesh-free FEM is suitable for electromagnetic field problems with large geometrical variations.

***Index Terms*— Electromagnetic field, finite element method, large geometrical variation, overlapping, remesh-free.**

I. INTRODUCTION

IN computational electromagnetics using finite element method (FEM), one fundamental work is to obtain the field distribution for the problems defined on a domain which contains several objects, and these objects may be complex and have time-dependent geometry[1]. In this case, the mesh generation at each time step in transient FEM may be required and may cost dramatic amount of computing time. One of the approaches to solve this type of problems is to develop a mesh structure which can make the mesh regeneration much convenient or the mesh regeneration is not required. One of such techniques is to allow overlapping meshes where a mesh of an object is allowed to overlap a background mesh representing the surrounding of the object.

The technique of overlapping meshes can significantly simplify the modeling using FEM and provide a generalized framework for FEM simulations. It is of particular interest in simulations which involve moving objects or large dimension changes during optimization. The main advantage is that, by using the overlapping mesh technique, one can avoid large deformation of the meshes. The deformation may lead to deterioration of the mesh quality. Therefore, it is much better than traditional mesh regeneration method.

The overlapping method was first proposed by Heinz Kreiss. Then more detailed discussions of the mesh generation can be found, for example, in the book of Thompson et al. [2]. After that, Chesshire and Henshaw [3], Yu [5] also worked on similar problems. Many of the presented algorithms and the tools developed are of interest for the implementation of various overlapping mesh techniques.

The overlapping mesh method has been also widely introduced into the study of fluid mechanics. The coupling at the fluid-fluid interface between the overlapping and underlying fluid meshes is handled using a stabilized Nitsche method developed for the Stokes problem [6]. One can also consider the problems where the structure is described via its moving boundary which is immersed into a fixed background fluid mesh. The classical immersed boundary method was introduced by Peskin [7]. Such hybrid schemes are built upon the concept of overlapping meshes introduced for finite differences and finite volume schemes in the early works of Starius [8].

In this paper, a method of two layers of overlapping meshes is presented. Instead of re-mesh the entire solution area as in traditional FEM, all meshes will remain the same in the proposed technique. It has the advantage of simple element assembly and is easy to be implemented in computer program. Numerical tests are be executed to showcase that the proposed methodology is practical.

II. METHODOLOGY OF REMESH-FREE FEM

The overlapping algorithm deals with the problems on the domain $\Omega_0 = (\overline{\Omega_1 \cup \Omega_2})^\circ$ in R^2, with boundary $\partial\Omega_0$, consisting of two (open and bounded) sub-domains Ω_1 and Ω_2 separated by the interface $\Gamma = \partial\Omega_1 \cap \partial\Omega_2$ .We consider the following elliptic model problem as shown in Fig.1: to find the $u : \Omega_0 \to \mathbb{R}$,

$$-\Delta u_i = f_i \quad \text{in } \Omega_i \quad i = 1,2 \tag{1}$$

$$[\nabla u \cdot \mathrm{n}] = 0 \quad \text{on } \Gamma \tag{2}$$

$$u = 0 \quad \text{on } \Gamma \tag{3}$$

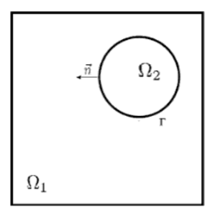

Fig.1. Two domains are separated by the boundary.

Implementing the overlapping mesh method has the following steps.

The first step is to mesh both the background and the objects separately.

The second step is to determine which cells are involved in the intersection between the two meshes. A proper searching area on the background mesh is identified. In this part, we introduce the displacement method to accomplish the collision detection.

The third step is element assembly, which is to compute the contributions from the cut cells of the background mesh.The intersection interface is represented by polygons.

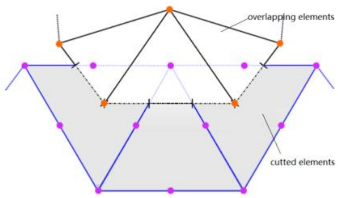

Fig. 2. The overlapping meshes in the intersection zone.

As shown in Fig. 2, this computation may be phrased in terms of so-called Boolean operations which are widely used to build complex geometries by performing Boolean operations between primitives from a finite set of geometries. The information of cut elements includes the number of vertexes, number of elements, boundary conditions and so on. After all information is determined, it will be stored and prepared for the next step.

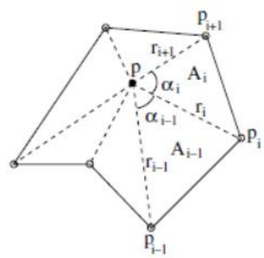

Fig. 3. Mean value coordinates construction.

The last step is to compute the FEM with polygon elements and complete the post-processing. An interpolation scheme for a scalar-valued function $u(\mathrm{x}): \overline{\Omega} \rightarrow \mathbb{R}$ can be written as:

$$u(\mathrm{x})=\sum_{i=1}^{n}\varphi_i(\mathrm{x})u_i \tag{4}$$

$$\varphi_i(\mathrm{x})=\frac{w_i(\mathrm{x})}{\sum_{j=1}^{n}w_j(\mathrm{x})} \tag{5}$$

$$w_i(\mathrm{x})=\frac{\tan(\alpha_{i-1}/2)+\tan(\alpha_i/2)}{r_i} \qquad r_i=|\mathrm{x}_i-\mathrm{x}| \tag{6}$$

where $r_i = r_i(\mathrm{x})$ is the Euclidean distance between p and p_i, and the angle α_i is shown in Fig. 3.

The entire mesh system is divided into three types of elements: background elements, cut elements, overlapping elements. The elements on different area are assembled separately.

III. Numerical Test

In order to verify that the proposed FEM is practicable for electromagnetic problems, a simple electromagnetic field problem is tested.

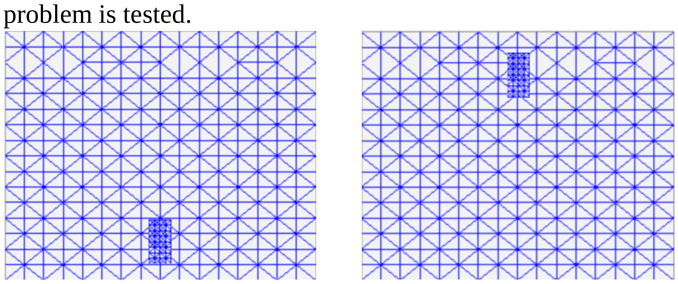
(a)Initial position. (b) End position.

Fig. 4.The overlapping meshes for the electromagnetic field problem.

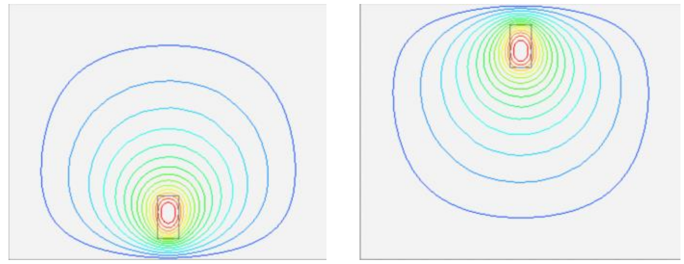
(a)Initial position. (b) End position.

Fig. 4. Flux lines of the electromagnetic field.

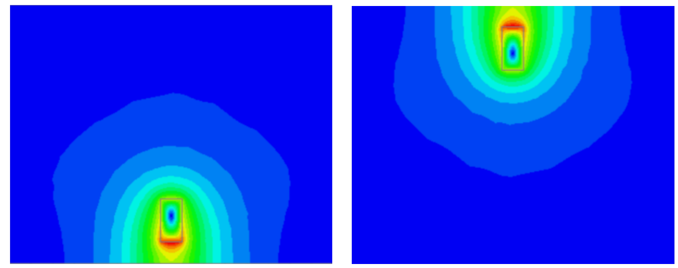
(a)Initial position. (b) End position.

Fig. 4.Distribution of magnetic flux densities.

TABLE I
COMPARISON OF REMESH-FREE FEM TO TRADITIONAL FEM

Methodology	Times of mesh generation	Meshing time(s)	Working time(s)
Traditional FEM	10	3.1	23.7
Overlapping remesh-free FEM	1	0.1	16

IV. Conclusion

The aim of this study is to develop a new methodology of FEM which can avoid remeshing when the dimensions of objects have large variations. The numerical experiment shows that the proposed overlapping remesh-free FEM is capable of computing electromagnetics field problems correctly. The comparison of the results between proposed methodology and traditional FEM shows that the proposed method is able to avoid mesh regeneration.

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